A new method of analysis of associative algebras

Vladimir Dergachev

volodya@mindspring.com

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \Leftrightarrow$$

	a	b	c	d
a	a	b	0	0
b	0	0	a	b
c	c	d	0	0
d	0	0	c	d

Fix F, study $\{a\}$:

$$a : \forall x \Rightarrow F(ax) = \alpha F(xa)$$

Better:

Let A be the multiplication table. Study $\lambda\, A|_F + \mu\, A^T\big|_F$

Spaces $V(\lambda, \mu) = \ker\left(\lambda\, A|_F + \mu\, A^T\big|_F\right)$

- are not generally ideals or even subalgebras
- simplify multiplication table A
- $V(\lambda_1, \mu_1) \cdot V(\lambda_2, \mu_2) \subset V(\lambda_1 \lambda_2, \mu_1 \mu 2)$
- $V(1, 0) \cdot V(0, 1) \subset V(1, 0) \cap V(0, 1)$
- can be generalized to Jordan spaces
- $\dim V(\lambda, \mu) = \dim V(\mu, \lambda)$

Example: multiplicative functionals

Algebra $\mathfrak{A}$ with unity. F is multiplicative iff $F(1) = 1$ and

$$F(ab) = F(a)F(b)$$

Multiplicative functionals, contd.

F is multiplicative if and only if $A|_F$ has rank 1 and $F(1) = 1$.

Proof:

- Consider $V(1, 0)$ and $V(0, 1)$.
- We must have $V(1, 0) = V(0, 1) = \ker F$
- Thus $\ker F$ is an ideal

This proof is valid for both finite and infinite dimensional algebras.