Orbit method on associative algebras

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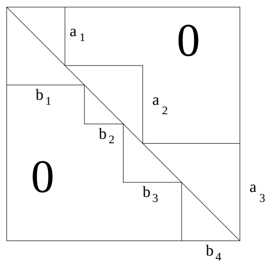
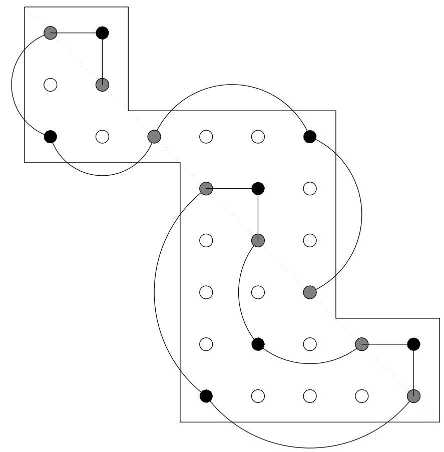
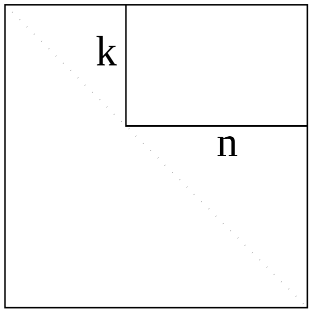


Answer:



What about



Index $= \gcd(k, n)$

"Easy" problem:

Prove that $\mathrm{ind}\,(\mathrm{Mat}_n \otimes B) = n \cdot \mathrm{ind}\, B$

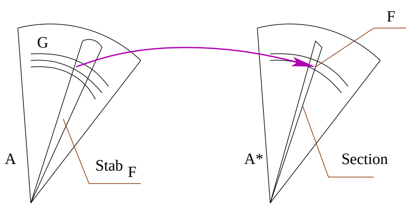
Generally:

$$\mathrm{ind}\,(A \otimes B) \stackrel{?}{=} \mathrm{ind}\, A \cdot \mathrm{ind}\, B$$

Why study $\mathrm{ind}\, A$?

- Orbit method: $\dim A - \mathrm{ind}\, A$ is the maximal possible dimension of a coadjoint orbit.
- A possible functor:
  $\mathrm{ind}\,(A \oplus B) = \mathrm{ind}\, A + \mathrm{ind}\, B$
  $\mathrm{ind}\,(A \otimes B) \stackrel{?}{=} \mathrm{ind}\, A \cdot \mathrm{ind}\, B$
- Why not ?

Orbit method picture



- $G = \mathrm{Auto}\ A$
- Stab_F is commutative for generic F
- Stab_F acts on F - image is a section
- Is Stab_F a Frobenius algebra ?

Type 1 algebras

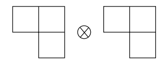
- $Q(x,y) = F(xy)$ for $x, y \in \mathrm{Stab}_F$
- Type 1 algebra: symmetric form $Q(x,y)$ is non-degenerate for generic F (i.e. Stab_F is Frobenius).
- Example: unital algebra with $\mathrm{ind}\, A = 1$

Theorem. Let A be a type 1 algebra, then

$$\mathrm{ind}\,(\mathrm{Mat}_n \otimes A) = n \cdot \mathrm{ind}\, A$$

Is it true that $\mathrm{ind}\,(A \otimes B) = \mathrm{ind}\, A \cdot \mathrm{ind}\, B$?

Counterexample:

ind () = 3

Both are type 1 algebras !

What is happening ?

- Define $\mathrm{Stab}_F(\alpha) = \ker\,(F(xy) - \alpha F(yx))$
- $\mathrm{Stab}_F(\alpha) \otimes \mathrm{Stab}_G(\beta) = \mathrm{Stab}_{F\otimes G}(\alpha\beta)$
- $\alpha = \alpha(F)$ can depend on F !

Conjecture:

$$\mathrm{ind}\, A \otimes B = \mathrm{ind}\, A \cdot \mathrm{ind}\, B + \sum_{1 \neq \alpha \in H} \dim \mathrm{Stab}_F^A(\alpha) \cdot \dim \mathrm{Stab}_G^B(\frac{1}{\alpha})$$

Here H denotes constants α common to both A and B.

References

- I.M.Gelfand and A.A.Kirillov, *Sur les corps liés aux algébres enveloppantes des algébres de Lie*, Publications mathématiques **31** (1966) 5-20
- Vladimir Dergachev and Alexandre Kirillov, *Index of Lie algebras of Seaweed type*, *Journal of Lie theory* **10** No. 2 (2000), 331-343
- Vladimir Dergachev, *A non radical based approach to study of associative algebras*, math.RA/0212369